[4] Potter, J L and Whitfield, J D, "Effects of slight nose bluntness and roughness on boundary-layer transition in supersonic flows," J Fluid Mech **12,** 501–535 (1962)

[5] Braslow, A L, "Review of the effect of distributed surface roughness on boundary-layer transition," AGARD Rept 254 (April 1960)

[6] Klebanoff, P S, Schubauer, G B, and Tidstrom, K D "Measurements of the effect of two dimensional and three dimensional roughness elements on boundary layer transition,' J Aeronaut Sci **22,** 803–804 (1955)

Reply by Authors to J L Potter and J D Whitfield

James R Sterrett* and Paul F Holloway†
NASA Langley Research Center, Hampton, Va

Nomenclature

k = vertical height of roughness above plate, ft
$\dot{q}$ = heat flow rate along the centerline of the model
R_0 = freestream unit Reynolds number per ft
R_k = Reynolds number based on fluid conditions at top of the roughness elements and the height of the roughness
R_k = roughness Reynolds number for which a further increase in roughness height causes no appreciable forward movement of the beginning of fully developed turbulent flow
x = distance from the leading edge
x_k = distance from the leading edge to the roughness position

THE salient points mentioned in the preceding comments were not discussed in Ref 1 because of a desire to make that manuscript as brief as possible However, these matters (including references of the preceding comment) are discussed in a more detailed report,[2] which was under preparation at the time the original manuscript was submitted The authors certainly feel that any full length paper on transition should include a reference to the work of Potter and Whitfield

The discrepancy between the Reynolds numbers for natural transition in Figs 1 and 2 of Ref 1 is thought to result not only from small variations in the leading-edge thickness, but also from a small angle-of-attack variation between the two assemblies However, the angle of attack for each series of roughness tests was invariant The interchangeable leading-edge section method of varying the roughness elements has some inherent disadvantages since leading-edge thickness is a factor in determining the location of transition However, this method was chosen as a practical means of reducing the required time for a model change

Since this early work indicated that small roughness heights may delay transition, further research on this phenomenon is being conducted (This work has been further stimulated by the comments of Potter and Whitfield) Some new data now available have indicated that whereas some of the variation in the location of transition reported in Ref 1 was due to variation of the leading-edge thickness, under certain conditions transition is apparently slightly delayed when the surface roughness is less than the boundary layer thickness Figure 1 presents the heat flow rate distribution for the model with various height roughness elements at a unit Reynolds number of approximately 8 × 10^6/ft and a Mach number of 6 Test conditions are similar to those given in Ref 1 A schematic of the model shown in the figure illustrates the new

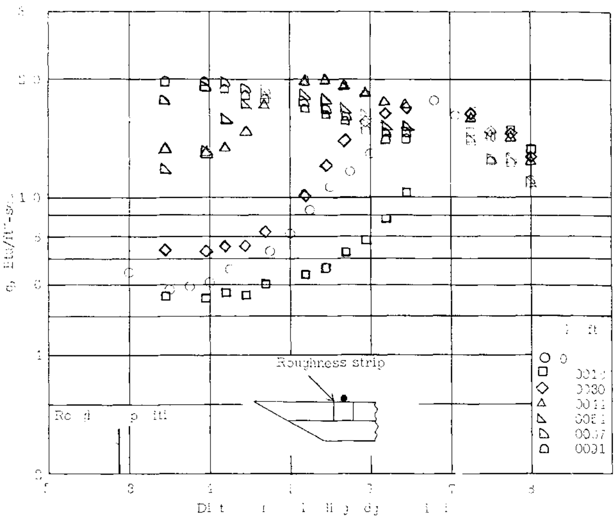

Fig 1 Heating rate distribution along flat plate with various size roughness, R_0 = 8 3 × 10^6

mounting technique with the same leading edge (thickness diameter of 0 0025 in) being used for all tests and the small roughness strips being interchangeable This figure illustrates both the delay in transition obtained with the smallest roughness elements and the critical roughness height required for these freestream conditions (critical height is taken as 0 0091 in the figure) Figure 2 shows the variation of the critical roughness Reynolds number with freestream unit Reynolds number Included in this figure are the data from Fig 3 of Ref 1 and new data taken with one leading edge with roughness elements mounted on an interchangeable strip Although the new and old data do not coincide, which may be due in part to a different location (x_k) of the roughness elements, these data do not change the conclusions given in Ref 1 A study of the effect of small roughness heights on transition is being continued

The definition and determination of the critical roughness Reynolds number varies considerably in the literature Difficulties arising from this were recognized, and as a result the roughness data were also compared in Ref 2 to other transition roughness data detected by a method sensitive to permanent changes in the boundary layer This comparison did not change the conclusions given in Ref 1

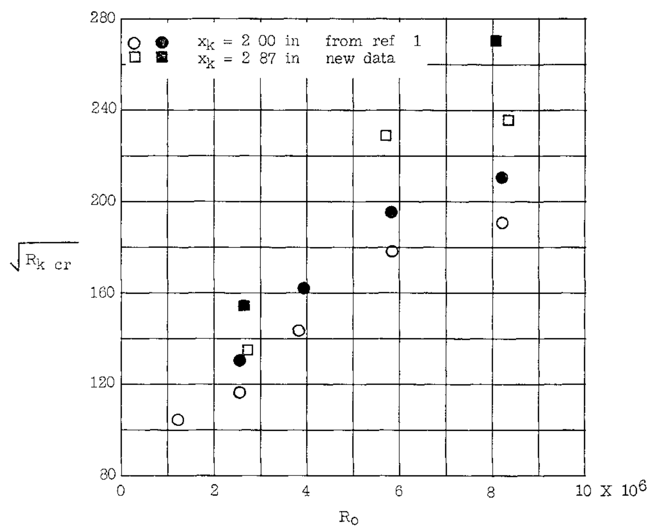

Fig 2 Variation of critical roughness Reynolds number with freestream Reynolds number Open symbols indicate that the roughness height is slightly less than the critical value, and solid symbols indicate that the roughness height is slightly greater than the critical value

Received November 20, 1963

* Head, 20-in Hypersonic Tunnel Section, Aero Physics Division
† Aero-Space Engineer Aero Physics Division Member AIAA

Refei ences

[1] Sterrett, J R and Holloway, P F, "Effects of controlled roughness on boundary-layer transition at a Mach number of 6 0' AIAA J 1, 1951–1953 (1963)

[2] Holloway, P F and Sterrett, J R, "Effects of controlled surface roughness on boundary layer transition and heat transfer at Mach numbers of 4 8 and 6 0" Proposed NASA TN D-2054

Comment on "Interception of High-Speed Target by Beam Rider Missile"

N X Vinh*

University of Colorado, Boulder, Colo

IN Ref 1, it is possible to express the missile coordinates explicitly in terms of the normal elliptic integrals F and E

The linear differential equation considered by the authors was

$$(d/d\gamma)(\cot\theta) + \tfrac{1}{2}\cot\gamma\cot\theta + \tfrac{1}{2} = 0 \tag{1}$$

Let ds_m be an element of arc of the missile trajectory and

$$ds_m = V_m dt = (V_m/V_t)V_t dt = \tau R d(\cot\theta)$$

With $\tau R = c$ and $\cot\theta = x_m/y_m$, we have,

$$ds_m = cd\left(\frac{x_m}{y_m}\right) = c\left(\frac{dx_m}{y_m} - \frac{x_m dy_m}{{y_m}^2}\right) \tag{2}$$

Since

$$dx_m = \cos\gamma ds_m \qquad dy_m = \sin\gamma\, ds_m$$

Eq (2) becomes

$$ds_m = c\left(\frac{\cos\gamma}{y_m} - \frac{x_m \sin\gamma}{{y_m}^2}\right) ds_m \tag{3}$$

Therefore,

$${y_m}^2 = c(y_m\cos\gamma - x_m\sin\gamma) \tag{4}$$

This equation clearly shows that the velocity of the missile is tangent to the circle centered at the origin and of radius ${y_m}^2/c$, a result mentioned in Ref 2 using different arguments

Therefore,

$$\cot\theta = \frac{x_m}{y_m} = \cot\gamma - \frac{y_m}{c\sin\gamma} \tag{5}$$

Using Eq (5) in (1), one obtains the differential equation

$$2\sin\gamma(dy/d\gamma) - \cos\gamma y + 1 = 0 \tag{6}$$

which is also linear with $y = y_m/c$

Integrating Eq (6) yields

$$y = \cos\phi\left(C + \frac{F(\phi,k) - 2E(\phi,k)}{2^{1/2}}\right) + \sin\phi(1+\cos^2\phi)^{1/2} \tag{7}$$

and using (4) we obtain the x coordinate as

$$x = -\left(C + \frac{F-2E}{2^{1/2}}\right)\times \left[C + \frac{F-2E}{2^{1/2}} + \tan\phi(1+\cos^2\phi)^{1/2}\right] \tag{8}$$

where F and E are normal elliptic integrals of the first and second kind with moduli $k = 1/2^{1/2}$ and arguments $\phi = \text{arc}\cos(\sin\gamma)^{1/2}$

The constant C is determined by the initial conditions We also have

$$\cot\theta = \frac{x}{y} = -\frac{1}{\cos\phi}\left(C + \frac{F-2E}{2^{1/2}}\right) \tag{9}$$

which is Eq (9) in Ref 1

Using the initial conditions, when $x = y = 0$, $\theta = \theta_0 = \gamma_0$, we have for the constant C,

$$C = \frac{2E(\phi_0) - F(\phi_0)}{2^{1/2}} - \tan\phi_0(1+\cos^2\phi_0)^{1/2} \tag{10}$$

By these expressions it can easily be seen that:

1) $y = 1$ is an asymptote since x becomes infinite for $\gamma = 0$ except when

$$C - \frac{2E(\pi/2) - F(\pi/2)}{2^{1/2}} = 0$$

or $C = 0\ 599$ This gives $\theta_0 = 37°$

2) The points where the velocity is directed vertically are such that $\gamma = \pi/2$, $\phi = 0$ Therefore $x = -C^2$ and $y = C$ Hence, they are situated on the parabola $y^2 = -x$, a result also mentioned in Ref 2 using different arguments

References

[1] Elnan, O R S and Lo, H, Interception of high-speed target by beam rider missile," AIAA J 1, 1637–1639 (1963)

[2] Wilder, C E, "A discussion of a differential equation,' Am Math Monthly 38, 17–21 (1931)

Received September 18, 1963

* Colonel Chief of Staff (1957–1962), Vietnamese Air Force; now Graduate Student Department of Aerospace Engineering

"Equilibrium" Gas Composition Computation with Constraints

Kenneth A Wilde*

Rohm & Haas Company, Huntsville, Ala

IT has been suggested in a recent note[1] that one may "freeze" a particular species in a chemical equilibrium computation by introducing a fictitious second set of species and manipulation of the real and imaginary sets of species, within the usual procedures of the computation routine The purpose of this note is to point out that such a constraint may be imposed on the equilibrium calculation in another manner when one is using the popular minimization of free energy technique [2] In this approach the Gibbs free energy is minimized subject to the constraints of the mass balances

$$\sum_{i=1}^{n} a_{ji}x_i = b_j \tag{1}$$

where a_{ji} is the number of atoms of element j in species i, x_i is the moles of species i per unit mass, and b_j is the number of gram-atoms of element j per unit mass

It is possible, however, to impose *a priori* relations (constraints) among the x_i composition variables by considering the constraints to be pseudo elements, as long as the relations are of the form of Eq (1), i e, linear Thus the formula matrix a_{ji} would have a column for each species and m rows for each of the m real chemical elements, as usual, with an additional row for each constraint Possible constraints

Received October 11, 1963 This work was carried out under Contract No DA-01-021 ORD-11878(Z) Mod No 7

* Senior Research Chemist, Physical Chemistry Group, Redstone Arsenal Research Division